Design of a 3 Span Double Track Reinforced Concrete Railroad Arch Bridge

C. S. Millard G. A. Haggander

1907

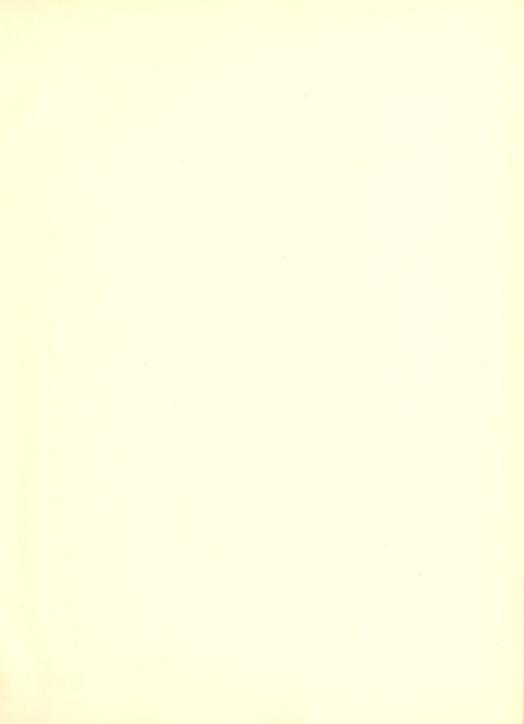
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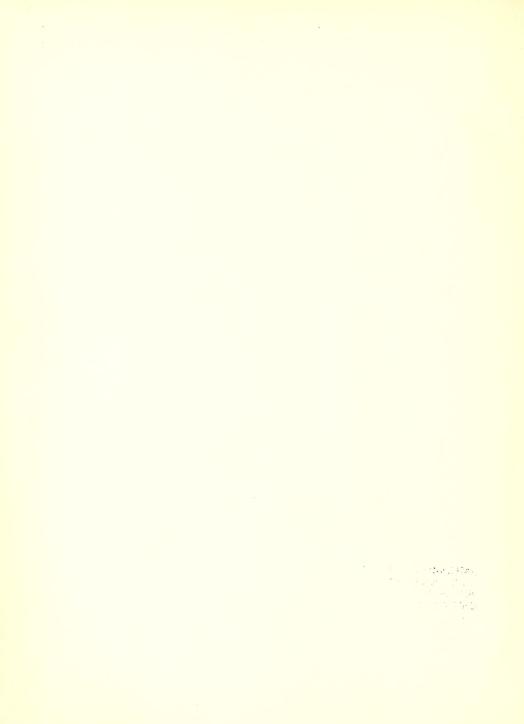
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DESIGN OF A THREE SPAN DOUBLE TRACK REINFORCED CONCRETE RAILROAD ARCH BRIDGE

A THESIS PRESENTED BY

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to the

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Chicago, Ill., June 1st, 1907.



The bridge, as designed, is a proposed structure for the C. & N. W. R. R. across the North Branch of the Chicago River, about four miles above the head of navigation. At the present time a wooden pile trestle carries the railroad across a rather shallow wooded valley. This valley of the upper waters of the North Branch has been under consideration, by various improvement associations, as a desirable link in an Outer Park System for the city of Chicago. With this fact in mind, the bridge was provided with two forty foot arches to accommodate future driveways.

METHOD OF DESIGN.

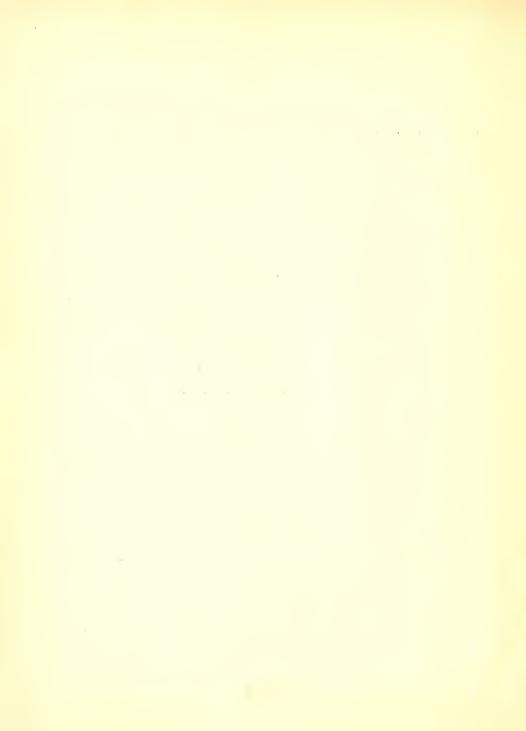
The method of design used in the analyses of the arches, was the graphical system developed by Mr. Burton R. Leffler, Bridge Engineer for the L. S. & M. S. R. R., in his treatise on "The Elastic Arch." In indicating the method of design, the calculations for one of the forty foot arches will be explained.

THE FORTY FOOT ARCH.

Two assumptions must be made to start with, (1) the thickness of the arch ring at the crown, (2) the curve of the intrados.

The thickness of the ring at the crown was assumed to be 2 feet,
the curve of the intrados was struck from 3 centers.

The theory is based on an equal number of horizontal divisions of the arch ring. The divisions were made on the dotted ordinates with an ordinate at the center. See Plates I, II and III. This determines $\frac{dl}{T}$ at the crown, where dl is the length of a division measured along the gravity axis, which is yet undeter-



mined. For a trial value, dl is measured along the intrados. I is taken at the center of each division of the gravity axis, assuming a width of arch ring of 1 foot. The remainder of the arch ring must be taken so that $\frac{dl}{l}$ is a constant.

$$\frac{d1}{I} = \frac{d1'}{I'}$$

Where dl' and I' are taken at the springing of the clastic archidl' = 2.33 dl = 2 I = $\frac{bh}{12}$ I' = $\frac{b}{12}$ $\frac{h'}{12}$

h = depth of the ring at the springing.

$$h' = \frac{2.35 \times 12}{.666} = 2.1$$

The span of 40 feet is not the true span of the elastic arch. The span of the elastic arch lies between the points where the tangents to the arch are fixed in direction. These points may be located at that portion of the ring where a sudden enlargement of section takes place, or where $\frac{d1}{1}$ ceases to be a constant. In the arch under consideration, this length of span was taken as 32 feet.

LOADS.

The live load was taken as the equivalent uniform load for Cooper's E60 with 100% impact added. The load was considered to be distributed over a width of 12 feet. From Cooper's Specifications for Railroad Bridges, the equivalent uniform load for E40 with a span of 32 feet = 7120# per foot. For E60 this equals $\frac{6}{4}$ x 7120 = 10680#. With 100% impact added this equals 21360# per foot. In designing the arch, for convenience, a section 1 foot



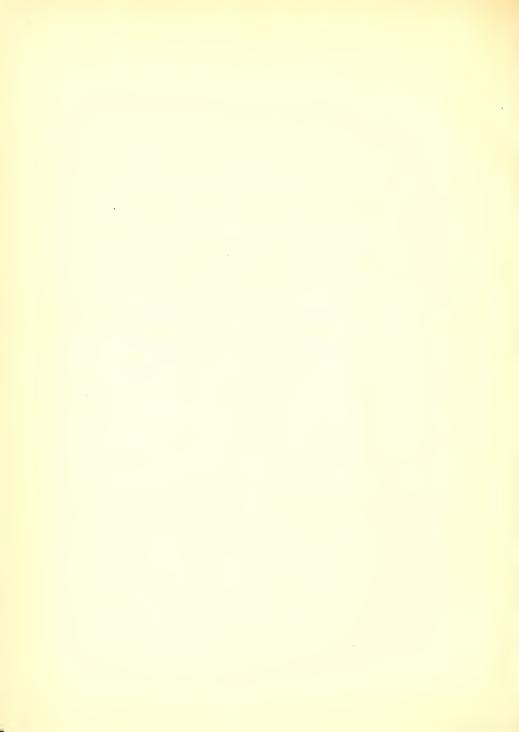
wide was used. The load on a width of one foot = 1780#. 1800# per linear foot was the load used. The volume of the dead load was scaled from the drawing. The weight of the earth fill was taken as 100# per cubic foot, and the weight of the arch ring 150# per cubic foot. The volume of fill at section $0 = 1.5 \times 2 = 3$ cubic feet. The weight of earth = $3 \times 100\# = 300\#$. The weight of the arch ring = $2 \times 2 \times 150 = 600\#$. The total load at the crown = 900# + 3600# = 4500#. The length of a division was taken as 2ft and therefore the live load per section = 3600#.

THE DETERMINATION OF THE TRUE VALUE OF "H."

In order to determine the maximum stresses at the various sections, at least three positions of the live load rust be considered. The positions used were 1/2, 3/4 and full load. Taking the position of half load as an example, the load line was laid off vertically, each load being numbered corresponding to its position along the arch. Assuming H, the horizontal thrust as 40000#, a trial stress diagram and equilibrium polygon was constructed.

The next portion of the problem, was to locate a line m m₁ (See Plates) so that the sum of the ordinates, called # ordinates, from this line to the trial equilibrium polygon, would be zero.

All ordinates were measured to the same scale as that of the arch, 3/8 inch = 1 foot. The method of determining m m₁ can better be understood by referring to the table on Plate I. Column 1 contains the number of the section. Columns 2 and 3 give the ordinates



nates to the trial equilibrium polygon on the right and left sides of the center; 4 gives the difference between the right and left ordinates of the same section; 5 gives the summation of these differences.

This summation was used in the formula for determining v w

$$n_{W} = \frac{12(d_1 + 2d_2 + 3d_3 + - - - 7d_7)}{(m+1)(m+2)}$$

Where n equals the number of equal spaces that the arch was divided into. To determine the direction from m m, v w was laid off above v and v, w drawn. R, the sum of the ordinates in columns 2 and 3 of the table = 48.95.

The formula $\frac{R}{n+1}$ gives the distance that m m, is above v, w at the mid-ordinate of the polygon.

$$\frac{R}{n+1} = \frac{48.95}{17} = 2.88$$

Having drawn m m, parallel to v, w, the m ordinates were measured. These ordinates were recorded in columns 6 and 7.

The y ordinates are given in column 8 of the table. They were measured from the line joining the springing points o y, to the gravity axis of the arch ring. The line k k, was next drawn parallel to o y, and at a distance above it equal to $\frac{\sum y}{m+1}$. The line m m, cuts the polygon at E and E. Projecting these points upward to e and e, in the line k k, locates 2 points in the required pressure curve. The k ordinates were measured from k k, to the gravity axis of the arch. The summations $\sum my$ and $\sum ky$ recorded



in columns 12 to 14, are the sums of the products of m, y and k.

Fo determine the true pole distance, the formula $\sum \frac{\mathbf{h} \mathbf{v}}{\mathbf{k} \mathbf{y}} \mathbf{x}$ Trial Pole Distance is necessary. If the true value of H was assumed in the first place, then $\sum \mathbf{v} \mathbf{y} = \sum \mathbf{k} \mathbf{y}$.

True Pole Distance = $\frac{32.09}{27.25}$ x 40000 = 47100[#].

To locate the true pole P, P, r was drawn parallel to m m,,
P then lies on a horizontal line through r.

In the fundamental equation $S = \frac{T}{A} + \frac{Mc}{T}$ S = stress, T the thrust, A the area, M the moment, c the distance to any fibre and I the moment of inertia of the section. In this analysis of the arch the effect of $\frac{T}{A}$ has been omitted.

By experiment it has been determined that the effect of $\frac{T}{A}$ is to decrease the value of H. The experiments of Prof. Howe have shown that for an arch having a rise of $\frac{1}{4}$ the span, the true value of H is 93-1/2% of the approximate value; for a rise of 1/6, 86%; and for a rise of 1/10, 69%. In the arch designed, the rise was 3 feet 10 inches, and the span 32 feet, giving a ratio of $\frac{1}{8.15}$. By interpolating between the values already mentioned the correct percentage for H was determined.

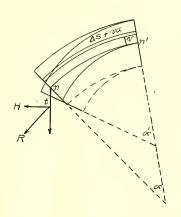
True Pole Distance = 72.6% of the approximate value.

Having determined the true pole distance, a new stress diagram and equilibrium polygon was drawn. e and e being points on the required curve, the new polygon must be drawn from one of these points, and as a check on the accuracy of the work, it should pass through the other point. The pressure curve having



been located, it only remains to determine the unit stresses.

UNIT STRESSES



The above figure is a side view of a portion of an arch ring contained between two planes perpendicular to the neutral surface n n' and making an angle \propto in circular measure, before strain, between them. A vertical plane midway between the faces of the arch intersects the neutral surface in the line n n' = Δs feet in length, which may be called the neutral line. The forces considered all act in this plane.

Let R be the resultant of all external forces acting upon the section passing through r. Conceivs applied at n two opposed forces +R and -R, each equal and parallel to R. The single force R is thus replaced by a couple RR and a force +R acting at n. The latter may be resolved into components T and N, tangential and normal to n n at n. The force T causes a uniform shortening in all the fibres. The force N is a shearing force and may be



neglected in determining the longitudinal stresses.

The couple RN is principally effected in changing the curvature of the arch and its moment is most conveniently found by multiplying its horizontal component H by the vertical distance from n to R, which we can call t feet. Then we have

M = Ht,in foot pounds (1).

Under the action of this couple the angle α is changed to α' and the curvature is increased if R cuts the section below n, and decreased when R cuts the section above n.

Call $\alpha' - \alpha = \Delta \alpha$ and regard M as right handed.

Call distance of any fibre from $n \ n' \ \nu$, this being \pm above and \pm below. The length of the fibre before flexure

 $=\Delta 5 + V\alpha$ after flexure $=\Delta 5 + V\alpha'$ its change of length is $V(\alpha'-\alpha') = V\Delta\alpha'$ Calling its cross section a in square feet and the unit stress due to M=1 founds per square foot, the stress on the fibre of concrete

$$fa = \frac{V\Delta\alpha}{\Delta S + V\alpha} \quad aE_{I} \tag{2}$$

and of steel

$$fa = \frac{V\Delta \propto}{\Delta s + V\alpha} an E, \tag{3}$$

since $f = \frac{\text{elongation of fibre}}{\text{length of fibre}} \times E$.

In (3) $n = \frac{1}{2}$ where E,= modulus of elasticity of concrete and $n = \frac{1}{2}$ modulus of elasticity of steel. $n = \frac{1}{2}$ can be replaced by $n = \frac{1}{2}$ without appreciable error. The sum of all the



stresses due to flexure on the entire section at n for concrete

$$\Sigma(fa) = \frac{E_1 \Delta \alpha}{\Delta 5} = \Sigma(Va) \tag{4}$$

or for steel

$$\Sigma(fa) = \frac{E_i \Delta \alpha}{\Delta S} = \Sigma(vna) \tag{5}$$

The moment of the stress (af) about n on any fibre is (afv)

$$M = \Sigma(afv) = E_1 \underbrace{\Delta \approx}_{\Delta 5} \Sigma(v^2 a)$$

Let I_r = moment of inertia of the concrete of area A_r in feet, and I_Z = moment of inertia of the steel of area A_Z in feet.

$$\sum V^2 a = \sum (V^2 a) \text{ for concrete}$$

$$= \sum (V^2 n a) \text{ for steel}$$

$$= I_1 + nI_2$$

$$\therefore M = E_1 \underbrace{\Delta \alpha}_{\Delta 5} (I_1 + nI_2)$$

$$\therefore \Delta \alpha = \frac{M\Delta S}{E_1(I_1 + nI_2)}$$

Let f, = stress per square inch on an extreme fibre of the concrete whose distance from the neutral axis is v, feet. Then from (2) $f_i = V_i E_i \Delta \propto \frac{\Delta}{\Delta S}$

and eliminating $\frac{\Delta \alpha}{\Delta s}$ between this equation and (6) We get

$$f_i = \frac{MV_i}{I_i + nI_2}$$

and for steel,

$$f_2 = \frac{MV2\Pi}{I_1 + \Pi I_2}$$



The direct thrust on the arch must now be found. Let P = the uniform compression on concrete of area A, and let up be the unit compression on steel of area A2.

Then the total compression on a section = $P(A_1 + \Pi A_2)$ This is equal and opposite to T.

from Which

$$P = \frac{1}{A_1 + nA_2} \qquad nP = \frac{nT}{A_1 + nA_2}$$

The total stress would now be the sum of the bending stresses and the direct thrust. Let s, and s₂ be the stress in pounds per square foot on the concrete and steel respectively at the upper and lower edges.

Then

$$S_{1} = \frac{T}{A_{1} + nA_{2}} \pm \frac{Mv_{1}}{I_{1} + nI_{2}}$$

$$S_{2} = \left(\frac{T}{A_{1} + nA_{2}} \pm \frac{Mv_{2}}{I_{1} + nI_{2}}\right)n$$

As an example take point 1 on the 80 foot arch.

Live load Thrust = 100,000#

Temperature Thrust = 15,600# T = 115,600#

Live load Bending Moment = 133,000 ft. pds.

Temperature " =
$$\frac{40,000}{173,000}$$
" " "



A, = 3 square feet

A = .0435 square feet

$$n = 15$$
, $I_1 = 2.25$, $I_2 = .0433 \times 1.25^2$

$$I_2 = .0433 \times 1.25^2$$

$$V_1 = 1.0$$
, $V_2 = 1.20$

$$\frac{S_l}{144} = \frac{115,600}{5 + 15 \times .0455} + \frac{173,000 \times 1.5}{2.25 \times .0455 \times 1.25}$$

-770 +333 pounds per square inch

$$\frac{\text{Sz}}{144} + \frac{115,600}{3+15 \times .0433} + \frac{173,000 \times 1.25}{-2.25+15 \times .0455 \times 1.25} = 15$$

-10,150 + 5,600 pounds per square inch.

TEMPERATURE STRESSES.

Let to = rise or fall of temperature in degrees Fahrenheit. D = span in feet

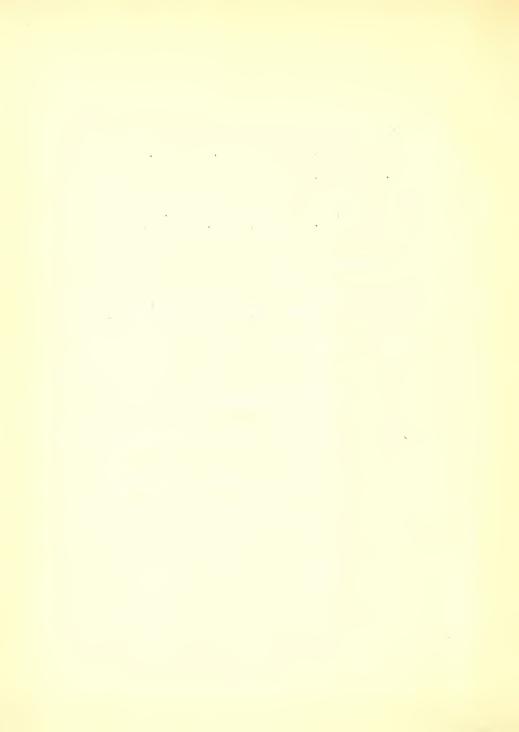
f = coefficient of expansion of concrete.

The lengthening or shortening of the span is Dtf.

This is the horizontal movement along the X axis and is also given by the formula

which is one of the equations leading up to the fundamental equations.

$$Dto f = \frac{Hell}{EI} \int ty$$
or
$$H = \frac{IDto fE}{dl \int ty}$$



Since the end tangents are fixed in direction $\sum t = 0$ which means that H acts along the line K K.

fty is the same as I fey which was found in computing the true pole distance.

The range of temperature was 28°. If the arch was built at 53° the range would be from 25° to 31°. The concrete is quite massive and is also covered with earth so that this range is undoubtedly sufficient.

As an example we will find H for the 80 foot arch.

$$I = \frac{9}{4} \qquad d1 = 4 \qquad D = 64$$

$$t = 28^{\circ} \quad E = 144 \times 2,000,000$$

$$fy = 102.76, \quad f = .0000055$$

$$H = \frac{9}{4} \times 64 \times 28 \times .0000055 \times 20000000 \times 144}{4 \times 102.76} = 15,600\#.$$

FOUNDATIONS.

The direct thrust of the 50 foot arch for full load is 125,000 pounds. The direct thrust of the 40 foot arch for no load is 48,000 pounds. The weight of the concrete and filling between the points considered in determining the above thrusts is 60,600 pounds. Combining these graphically and obtaining the resultant gave a thrust of 185,000 pounds acting diagonally through the center of the pier base. The vertical component of this is 172,000 pounds which must be taken by the foundation per



foot of width.

Considering the arch as 24 feet wide (since the load from each track was distributed over 12 feet) we get a total load on the foundation of 4,128,000 pounds or 2,064 tons. This load is taken by 98 piles giving a load of 21 tons on one pile. This hay seem excessive, but the impact which was included in the thrusts never reaches the piles, but is absorbed by the inertia of the filling and concrete.

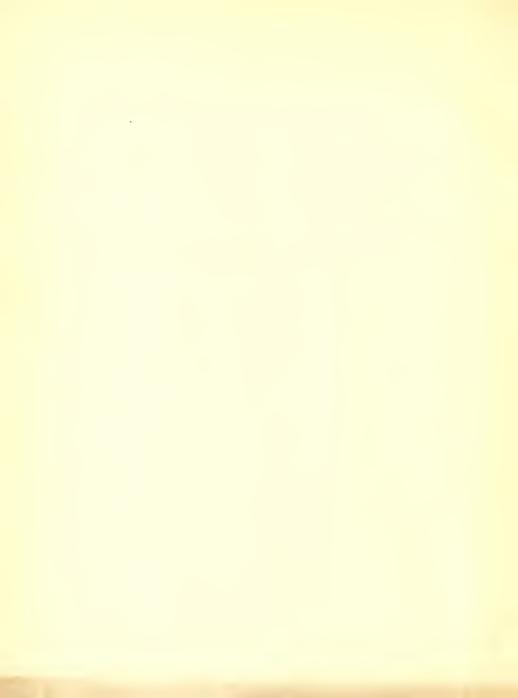
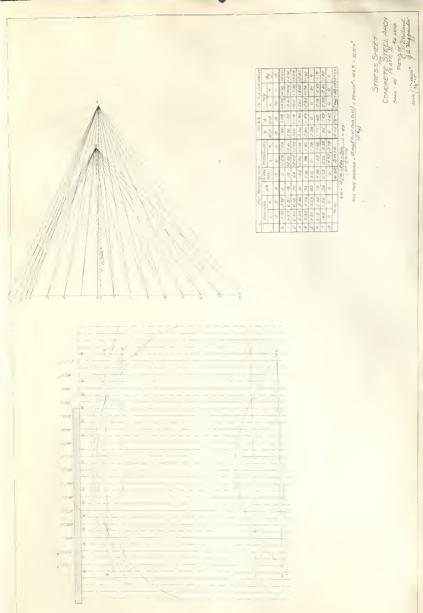


PLATE | For scale of original drawing multiply by 397







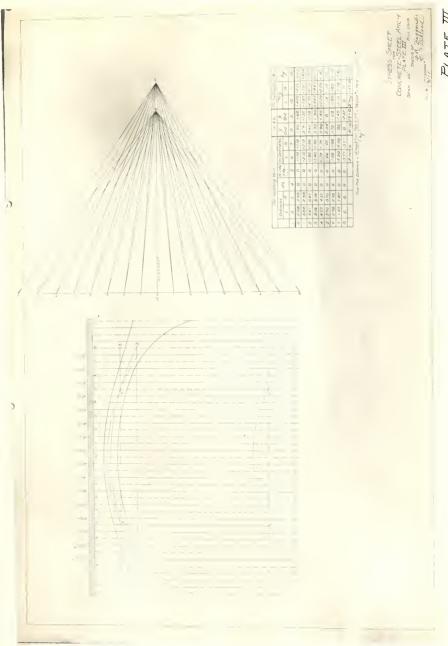


PLATE III For scale of original drawing, multiply by 4.09



PLATE IN For scole of original drawing multiply by 4.14

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\	5/.5	34.5	57.5	.66	.83	.50	34.4	28.7	28.7	53	74.9	57.4	Ŋ	-6.52	+1.14	-520	19/+
N	52.2	346	58	.58	. 9Z	.42	30.2	3/8	24.4	20.7	75.4	525	Ŋ	-6.22	t.73	-493	+ 132
D	53./	35	53	42	26.	.33	22.3	32.2	19.5	15.7	76.4	47.9	Q	-5.95	+.47	-467	+102
A	55.5	362	90	.25	.75	9/:	13.9	272	9.6	35	77.4	35.7	Q	- 5.17	04-	-395	+27
. p	55.7	38	615	9/	.25	0	0.0	9	0	0	78.9	9.5	Ŋ	-347	-2/9	-245	-/33
9	57.8	40	64	.33	.25	.25	2.6/	0/	9/	8///	81.4	3/	2:08	-5:01	+.85	-378	11+
7	90	42.6	99	.66	.83	.50	40	35.4	33	25.9	834	62:3	225	-6.85	+1.35	-535	197+
Ø	63	46	69	7.08	1.50	.83	68	72.7	572	4Z.1	86.4	114.2	325	-5.45	1/22	-462	+/85
600	W/A	JN.	Full	WA	10	Full	wla	19	Full	. 6	105	Ties	*	555	*	52	000
		Direct	to	4.	from &	Arch Ring		Moment	Foot Kips	Temp BM.	Max. Thrust-Kips	Maki Fr. K	Depth of	Mar. Stres	KIPS pera	Max Stre	Pounds pera + 185



		Ŋ	STRESS	S TABL	TE FOR		BOFT ARCH	CH		
7	7000	Q	7	9	6	4	W	7	\	0
+	WA	120.5	115.	110.5	101	104	1035	101	100.	100.
Thrust	Nr.	93.0	87.	825	79.	76.	74	72.5	72.	4 722
in Kips F	Full	123.0	118.	113.	109.5	106.5	1045	103.	102.	1017
4.4	w/A	1.42	.75	.25	.25	.75	1.00	1.25	1,33	. 133
	YN.	2.00	1.00	9/.	99.	1.17	1.50	1.67	1.63	1,211
Arch Ming	Fu//	1.75	1.08	.63	.08	.33	.63	.83	1.00	00%
00	mp	171.	86.3	276	26.8	78.	103.4	126.	133.	/33.
l lomen l	Nr.	186.	87.	13.7	52.7	88.7	///:	121	114	842
	Full	2/6.	128.	99	1.6	35.6	9	86.	102.	1017
Temp. BM. T= 15.6 Mps.		725	445	20.8	8./	14.3	26.6	36.6	40.	42.
Max. Mips	is	138.6	133.6	128.6	346	9%e //3%	.6//	116.6	115.6	115.6
Max. FIKIPS	sal	288.5	172.5	86.8	546	103.	130.	162	173.	175.
Depth of Ring - Feet		4.5	35	3.17	3.17	3.08	308	3.08	3.00	3.00
May.		-87	-8.7	-6,62	-456	-685	-83	-9.42	-1915	-1925
Steel Kips	5.0"	+3/	+2.05	755.−	65	+139	+163	+296	+36	+37
Mak		-747.	-640	-482	-329	-500	- 6/8	- 708	-770	-775
10 #	" 0	+ 299.	+193	+ 15.	- 12.	+150	+177	+278	+333	+338



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